Harold J. Larson

CONDITIONAL DISTRIBUTION OF TRUE RELIABILITY AFTER CORRECTIVE ACTION.

TR/RP NO. 61

UNITED STATES NAVAL POSTGRADUATE SCHOOL

CONDITIONAL DISTRIBUTION OF TRUE RELIABILITY

AFTER CORRECTIVE ACTION

by

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January 1966

Technical Report/Research Paper No. 61

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UNITED STATES MAVAL POSTGRADUATE SCHOOL
Monterey, California

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ABSTRACT

N items have been tested. Attempts are made to correct those failure modes that occur; the probabilities that these attempts are successful are assumed known. The conditional distribution of the resulting true reliability is derived and used to construct observable limits which will include the true reliability with a known probability.

This task was supported by: Special Projects, Code SP-114

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U. S. Naval Postgraduate School Technical Report, Research Paper No. 61
January 1966

UNCLASSIFIED

CONDITIONAL DISTRIBUTION OF TRUE RELIABILITY AFTER CORRECTIVE ACTION

1. Introduction

This paper considers a situation in which N items of a particular sort are tested. N_o of these items work correctly and $N-N_o$ fail. The items that fail are scrutinized and for some (or all) the cause of failure is determined. Corrective action then is taken to remove those failure modes that are observed in the N tests. The corrective actions taken may or may not be successful (in removing their respective failure modes from subsequent items); no further items of a corrected design are tested. It is assumed that the probability of correcting any observed failure mode is known.

The value of the true reliability of a system after attempts have been made to remove the failure modes that occur in N independent tests of the system is a quantity of vital interest. This true reliability is, of course, a random variable, a quantity whose value depends upon the outcome of a chance experiment. Since no tests of the system are assumed to have been made after the correctional attempts, there is available no direct experimental evidence on the reliability value finally attained. However, assuming we have available the results of the N tests of the system and that we know the probability that any occurring failure mode is corrected, it is possible to construct the (conditional) distribution of the true reliability, and to explore this distribution for possible meaningful statements that can be made.

The paper "Estimating Reliability After Corrective Action" defines p_o^* as a measure of current reliability and then goes on to discuss and compare estimators of the average value of p_o^*, called the mean reliability. It is stressed by the authors that the paper is concerned with estimators whose properties are to be studied prior to the N tests having been performed, thus justifying the concern about mean reliability, an average taken over all possible outcomes of the N tests. An approach such as theirs is

undoubtedly of interest in the early phases of development of particular types of item.

It would seem that an equally important problem can be phrased in a rather different way. Since the final estimator of achieved reliability (subsequent to the corrective action) is reasonably a function of the number of failures of various sorts that occur in the N tests, the conditional distribution of true reliability (conditional upon the outcome of the N tests) might be expected to yield some valuable information. This conditional distribution is exactly known (in some senses) at the time the actual estimator of achieved reliability is to be evaluated. This paper gives the conditional distribution of true reliability.

This conditional distribution of true reliability can be used to construct a conditional confidence interval for the achieved true reliability, based upon the results of the N tests. This conditional confidence interval gives a number, call it Z_o, such that the probability is at least $1-\alpha$ that the true reliability is as large as Z_o (or larger). As such, it reflects both the information contained in the results of the N tests and the assumed knowledge regarding ability to correct observed failure modes.

As will be seen, the conditional confidence interval statements that are given involve some generally described functions. It is expected, but not yet investigated, that for some very special types of these functions relatively tight probability statements can be made. That is, by adjusting the functions involved, it seems probable that the probability of exceeding Z_o might be made quite close to $1-\alpha$, rather than possibly being quite a bit larger than $1-\alpha$.

2. Conditional distribution of true reliability.

N systems are to be tested. Each system either operates successfully (with probability p_o) or it fails to operate successfully (with probability $1-p_o$). A system which does not operate successfully can fail according to any one of K different modes (with probabilities of occurrence equal

to q_i, $i=1, 2, \ldots, K$) .

$$\text{Thus } \sum_{i=1}^{k} q_i = 1-p_o$$

In writing the conditional distribution of true reliability, we can choose to represent either all K possible failure modes or to represent only those R modes that actually occur. If we make the first of these two choices, then in the N system tests we may not observe all K modes. Letting a_i be the known probability of correcting the i^{th} failure mode, $i=1, 2, \ldots, K$, the probability mass function of Z, the true reliability, is as given in Table 1, the variable x_i is 1 if the i^{th} failure mode is observed one or more times in the N tests and is 0 otherwise.

TABLE 1

z_i	$P(Z = z_i)$
p_o	$\prod_{i=1}^{k} (1-a)^{x_i}$
p_o+q_1	$a_1 x_1 \prod_{i=2}^{k} (1-a_i)^{x_i}$
p_o+q_2	$(1-a_1)\, a_2 x_2 \prod_{i=3}^{k} (1-a_i)^{x_i}$
$\vdots$	$\vdots$
p_o+q_k	$\prod_{i=1}^{k} (1-a_i)^{x_i}\, a_k x_k$
$p_o+q_1+q_2$	$a_1 x_1\, a_2 x_2 \prod_{i=3}^{k} (1-a_1)^{x_i}$
$\vdots$	$\vdots$
$p_o+q_{k-1}+q_k$	$\prod_{i=1}^{k-2} (1-a_i)^{x_i} a_{k-1} x_k\, a_k x_k$
$\vdots$	$\vdots$
$p_o+q_1+q_2+\ldots+q_k=1$	$\prod_{i=1}^{k} a_i x_i$

This probability mass function is a little unusual in that we are assuming we do not know the values the time reliability takes on, since they are functions of p_o and the q_i's but we do know the probabilities with which these unknown values occur.

We can easily derive the moment generating function for this distribution by examining the terms we get in expanding such a function as

$$(a_1+b_1)\,(a_2+b_2)\,\ldots\,(a_k+b_k).$$

If we form the sum defining

$$\psi_Z(t) = E(e^{tZ})$$

this sum contains all the terms (and no others) in the expansion of

$$e^{tp_o}\prod_{i=1}^{K}\left(a_i x_i e^{tq_i} + (1-a_i)^{x_i}\right).$$

Thus, the moment generating function for Z (in this formulation) is

$$\psi_Z(t) = e^{tp_o}\prod_{i=1}^{K}\left(a_i x_i e^{tq_i} + (1-a_i)^{x_i}\right)$$

Evaluating $\psi'_Z(0)$ and $\psi''_Z(0)$ we find

$$E(Z) = p_o + \sum_{i=1}^{K} a_i x_i q_i = p_o^* ,$$

$$E\left[\left(Z-E(Z)\right)^2\right] = \sum_{i=1}^{K} a_i(1-a_i)\,x_i q_i^2$$

(Both of these expectations are conditioned upon the results of the N tests). Thus, the measure of true reliability (p_o^*) mentioned in the paper by Corcoran, Weingarten and Zehna is in fact the mean value of the conditional distribution of final reliability. If we average this mean value

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[illegible]

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over all possible experimental outcomes, we arrive at mean reliability, the quantity they were estimating. Especially since the actual estimation takes place only after the N tests have been performed, it would seem reasonable to try to estimate p_o^* (or some other property of the conditional distribution) rather than something influenced by events that are known not to have occurred, when the estimation actually takes place. The second, simpler (and more natural) representation of the conditional distribution of true reliability is given by considering only those failure modes that actually occur in the N system tests (rather than considering all possible failure modes that could occur). Let Z be the true reliability (after the N tests and corrective action) and assume that R failure modes occur. Then the conditional probability mass function for Z is as given in table one above, where we set $x_i = 1$, for $i=1, 2, \ldots, R$, and set $x_i = 0$, for $i= R+1, \ldots, K$.

Then the moment generating function for Z (in this revised situation) is

$$\psi_Z(t) = e^{tp_o} \prod_{i=1}^{R} \left(a_i e^{tq_i} + (1-a_i) \right)$$

The mean and variance of Z are

$$E(Z) = p_o + \sum_{i=1}^{R} a_i q_i$$

$$E\left[\left(Z - E(Z) \right)^2 \right] = \sum_{i=1}^{R} a_i (1-a_i) q_i^2 \ .$$

We would like to know the actual value that the true reliability has taken on, after the corrective action has been completed; this value is a random variable whose conditional distribution is given above. Since

the actual value cannot be observed, the natural substitute to turn to is estimation of various properties of the conditional distribution of Z. The question of point estimation in general will be addressed first and then the idea of interval estimation of Z will be investigated.

3. Point Estimation

The most frequently estimated parameter of a probability distribution is undoubtedly its mean. Accordingly, if we were to make a point estimate of some property of the conditional distribution of Z, the most logical candidate to consider first might be the mean of the distribution. We have available the results of the N system tests for making such an estimate and thus could consider any estimator which is a function of $N_o, N_1, N_2, \cdots N_R$. The estimator

$$p_1 = \frac{N_o}{N} + \sum_{i=1}^{R} a_i \frac{N_i}{N}$$

is in fact an unbiased estimate of p_o^*, averaging over all possible outcomes of the experiment. However, since we are committed here to considering only properties of the conditional distribution of Z, it would be no more than reasonable to consider the properties of estimators averaging only over those experimental outcomes such that the distribution of Z remains unchanged. If we do not take only such conditional averages into account, then since the conditional distribution of Z may change so may the quantity to be estimated and we are effectively estimating something different from what was intended. The conditional distribution of Z remains unchanged so long as the failure modes observed are unchanged, no matter how many times they are observed. Thus we would like to average only over those experimental outcomes such that the observed failure modes still occur and we are led

to defining the event A (on the space of all possible outcomes in the N system tests) to be the set of outcomes such that $N_1>0$, $N_2>0, \ldots, N_R>0$, where failure modes 1, 2 ..., R are the ones that are observed in the tests. If A occurs, the conditional distribution of Z is fixed. In discussing properties of point estimators we shall make use of conditional expectations only, in accordance with the discussion just given.

As was mentioned above, the estimator

$$p_1 = \frac{N_o + \sum_{i=1}^{R} a_i N_i}{N}$$

is an unconditionally unbiased estimator of p_o^* (since $E(N_i) = Nq_i$). However, it is not a conditionally unbiased estimator, as will be shown below.

For simplicity, suppose that we perform N system tests and observe only 2 failure modes. Define

$$A = \{(N_o, N_1, N_2): N_1 > 0, N_2 > 0\}$$

and then

$$P(A) = \sum_{i=0}^{N-2} \sum_{j=1}^{N-1-i} \binom{N}{i\,j\,k} p_o^i q_1^j q_2^k \qquad (k = N - i - j)$$

$$= 1 - (p_o+q_1)^N - (p_o+q_2)^N + p_o^N .$$

Without a great deal of difficulty we find (again $k \equiv N-i-j$)

$$E\left(N_0 \mid A\right) P\left(A\right) = \sum_{i=0}^{N-2} \sum_{j=1}^{N-1-i} i \binom{N}{i\,j\,k} p_0^i q_1^j q_2^k ,$$

$$= N p_0 P(A) ,$$

$$E\left(N_1 \mid A\right) P\left(A\right) = \sum_{i=0}^{N-2} \sum_{j=1}^{N-1-i} j \binom{N}{i\,j\,k} p_0^i q_1^j q_2^k ,$$

$$= N q_1 \left\{1 - (p_0 + q_1)^{N-1}\right\} ,$$

$$E\left(N_2 \mid A\right) P\left(A\right) = \sum_{i=0}^{N-2} \sum_{j=1}^{N-1-i} k \binom{N}{i\,j\,k} p_0^i q_1^j q_2^k$$

$$= N q_2 \left\{1 - (p_0 + q_2)^{N-1}\right\} .$$

Then, combining the above results we find

$$E\left(p_1 \mid A\right) P\left(A\right) = p_0 P\left(A\right) + a_1 q_1 \left\{1 - (p_0 + q_1)^{N-1}\right\}$$

$$+ a_2 q_2 \left\{1 - (p_0 + q_2)^{N-1}\right\} ,$$

thus

$$E\left(p_1 \mid A\right) = p_0 + \frac{a_1 q_1 \left\{1 - (p_0+q_1)^{N-1}\right\} + a_2 q_2 \left\{1 - (p_0+q_2)^{N-1}\right\}}{1 - (p_0+q_1)^N - (p_0+q_2)^N + p_0^N}$$

It would appear to be a hopelessly complex job to add a quantity to p_1 to correct for its bias (in estimating p_0^* conditionally); possibly it could not be done. Since conditional unbiasedness seemed so difficult to achieve, the investigation of point estimators has initially stopped at

this point. More thought should be given to what parameters of the conditional distribution are of particular interest before much of an attempt is made to construct "good" estimators of them. In passing, in the case of R failure modes occurring, it is conjectured that

$$E\left(N_i \mid A\right)P\left(A\right) = Nq_i \left\{1 - \sum_{\substack{m=1\\ m\neq i}}^{R}\left(p_o + \sum_{\substack{j=1\\ j\neq m}}^{R} q_j\right)^{N-1} + \sum_{\substack{n=m+1\\ m\neq i\\ n\neq i}}^{R}\ \sum_{m=1}^{R-1}\left(p_o + \sum_{\substack{j=1\\ j\neq m\\ j\neq n}}^{R} q_j\right)^{N-1} - \dots + \left(-1\right)^{R-1}\left(p_o + q_i\right)^{N-1}\right\}$$

for $i=1, 2, \dots, R$.

These values can (possibly) most easily be derived by taking derivatives of P(A) with respect to the parameters $p_o, q_1, \dots q_R$ and solving the resulting system of equations for the conditional expectations of interest.

4. Interval estimation.

An apparently more fruitful avenue of investigation is that of interval estimation of the true reliability Z. Since we know the probability distribution of Z it should be easy in theory to find a point b such that

$$P(Z > b) \geq 1 - \alpha$$

and thus the interval $(b, 1)$ would constitute a $100(1-\alpha)$ % confidence interval for Z. The reason that we cannot immediately find the value b, of course, is we do not in general know $p_o, q_1, \dots, q_K$ and thus do not explicitly know the values (nor even the ranking of them) that the true reliability will take on. However, there are some interesting lines of reasoning that can be explored, leading almost to confidence intervals for Z (one approach leads exactly to confidence intervals of the above sort). We shall consider two separate cases, the first assuming no information whatsoever about $p_o, q_1, \dots, q_R$ and the second, assuming

that we have available a ranking in order of magnitude of $q_1, q_2, \ldots q_R$, determined independently from the current set of tests. Within each of these we shall discuss both conditional and unconditional confidence intervals for Z.

In the event that $p_o, q_1, q_2 \ldots, q_R$ are completely unknown, the values and ranking of values that the true reliability takes on are unknown. However, due to the definition of the probabilities of successes and failures occurring there is a natural partial ordering that can be exploited to construct conditional interval statements about Z. Thus, since $p_o \geq 0, q_1 \geq 0, \ldots, q_R > 0$, we know that

$$p_o < p_o + q_1 < p_o + q_1 + q_2 < \ldots < p_o + q_1 + \ldots + q_R$$

$$p_o < p_o + q_2 < p_o + q_2 + q_1 < \ldots < p_o + q_1 + \ldots + q_R\,,$$

and so on. In fact, it can be shown that the true reliability Z may take on any of 2^R (conceivably) distinct values if R failure modes occur. $p_o + q_i$ will precede $2^{R-1}-1$ of these, for all i, and is indeterminate with respect to 2^{R-1} of them. More generally, p_o plus r q_i's will succeed 2^r-1 values, precede $2^{R-r}-1$ values and is indeterminate with respect to the remaining $(2^{R-r}-1)(2^r-1)$ values.

This natural partial ordering can be used to make statements such as

$$P\left[Z \geq \min (p_o + q_1, p_o + q_2, \ldots, p_o + q_R)\right]$$
$$= P\left[Z > p_o\right] = 1 - \prod_{1}^{R} (1-a_i)\,,$$

$$P\left[Z \geq \min. (p_o+q_1+q_2, p_o+q_1+q_3, \ldots, p_o + q_{R-1}+q_2)\right]$$
$$\geq 1 - \prod_{i=1}^{R} (1-a_i) - \sum_{i=1}^{R} a_i \prod_{\substack{j=1 \\ j\neq i}}^{R} (1-a_j)\,,$$

etc. It if should happen that one of these right hand sides is equal to a confidence level of interest, then we almost have a confidence interval.

Almost, because we still do not know p_o and the q_i's and thus would have to estimate them in order to have some idea where the minimum value involved might fall. But in making use of such an estimate we would no longer have an exact probability statement about the interval containing Z.

However, if we think of the compound experiment consisting of the N system tests and then the corrective action, the reasonable probability measure to use for the compound experiment is constructed by multiplying together the measure that applied to the N system tests, times the conditional measure applicable to the corrective action taken (conditional only in the sense that the experimental outcome dtermines the corrective action to be taken). Using this multiplicative rule enables us to make statements such as

$$P\left[\min(p_o+q_1, \ldots, p_o+q_R) > f(N_o, N_1, \ldots, N_R),\ Z \geq \min(p_o+q_1, \ldots, p_o+q_R) \mid A\right]$$

$$= P\left[\min(p_o+q_1, \ldots, p_o+q_R) > f(N_o, \ldots, N_R) \mid A\right]$$

$$\cdot\, P\left[Z \geq \min(p_o+q_1, \ldots, p_o+q_R) \mid A\right],$$

where A is the event defined above. The results of the N system tests are available to evaluate $f(N_o, N_1, \ldots, N_R)$ occurring in the first of these two probability statements (this is just a conditional multinomial probability statement) while reasoning such as the preceding may be used to evaluate the second. Their product gives us a statement that is somewhat similar to a conditional confidence interval; this product can be used to construct a lower bound confidence interval as follows. Let g be a function of the unknown parameters, let f be a function of the observable random variables $N_o, N_1, \ldots, N_R$, and let A be the event that $N_1 > 0, N_2 > 0, \ldots, N_R > 0$. Then a conditional confidence interval for Z would be a statement such as

$$P(Z > f \mid A) \geq 1 - \alpha$$

The statement we are able to make is

$$P(Z > g,\ g > f \mid A) = P(Z > g \mid A)\ P(g > f \mid A)$$
$$= 1 - \alpha$$

Since the set relationship $\{Z > g,\ g > f\} \subset \{Z > f\}$ must hold, we can say

$P(Z > f \mid A) \geq P(Z > g,\ g > f \mid A)$ and thus get a lower bound on the probability that an observable interval contains Z.

Unconditional confidence intervals would possibly not be of much interest, for many of the same reasons given previously regarding expectations averaged over all possible experimental outcomes. Since we would not be constructing the interval, presumably, until after the N system tests are completed, we would be more interested in the conditional measure of Z than we would be in the unconditional measure. Even if we had the interest, it would appear to be impossible to construct an unconditional confidence interval for Z, since the probability that Z exceeds some function f of the observable random variables $N_o, N_1, \ldots, N_R$ is going to depend upon what values of the form p_o, p_o+q_1, p_o+q_2, etc., are exceeded by f; given no information about $p_o, q_1, \ldots, q_R$, we are not able to determine which of these values are and which are not exceeded by f. Thus we cannot logically derive an unconditional measure of the probability that Z exceeds f.

Suppose we are given that

$$q_1 > q_2 > \ldots > q_R.$$

Then we would find that again Z can take on any of 2^R (conceivably) distinct values, but now p_o+q_1 is indeterminate with respect to only $2^{R-1}-R$ of them, p_o+q_2 is indeterminate with respect to $2^{R-2}-R+1$ of them, etc.; in general, p_o+q_i is indeterminate with respect to $2^{R-i}-(R-i)-1$ of the other values, for $i=1,2,\ldots,R-1$. The position of p_o+q_R is completely

determined with respect to all the possible values. Similarly, we find that the position of $p_o + q_1 + q_2$ is indeterminite with respect to $2^{R-3} - \binom{R-3}{2} - (R-3) - 1$ values, $p_o + q_1 + q_3$ is indeterminate with respect to $2^{R-4} - \binom{R-4}{2} - (R-4) - 1$ values, and so on. The assumed ranking of the q_i's thus leads to a more definite ranking of the values of Z, which in turn would imply that we have a much wider choice of values for statements such as

$$P(Z > g \mid A),$$

where g is some function of the unknown parameters. Thus we could construct confidence intervals for a much larger choice of probabilities. Exactly the same type of reasoning as that outlined above would lead to lower bounds on the conditional probabilities that particular intervals cover Z. Unconditional statements would appear as impossible of attainment here as in the preceding case, for exactly the same sort of reasons.

5. Topics for further research.

The conditional multinomial probability statement mentioned in section 4 above may or may not be a simple matter to derive. The feasibility and explicit method of attaining such a statement should be spelled out in detail.

The assumption that the a_i's are known (the probabilities of removing observed failure modes) may not be realistic. The affects of these quantities being unknown should be detailed. In many cases it would seem plausible to assume $p_o, q_1, \ldots, q_K$ known and $a_1, a_2, \ldots, a_K$ unknown; the affects of this change should be investigated.

It might prove profitable to replace the probabilities of correcting observed failure modes by the reliability of the replacement part (or design change). That is, if a design change is made then a_i is essentially the probability that the reliability of the change is 1; $1-a_1$ is the probability

that the reliability is less than 1 (but no indication of how much less than 1). Rather than include the parameters a_i in the model it might prove better to use the parameters b_i, defined to be the reliability of the changed item or portion of the design. This would, of course, allow final reliability after corrective action to decrease rather than only to increase as in the present model.

The whole basic model for reliability growth might be better if structured in some different manner. The suggestion made in the last paragraph is one possible restructuring; undoubtedly many others could be brought to mind. The restructuring should keep in mind simplicity of approach, ease of computation and estimation and yet remain as realistic as possible.

References:

Corcoran, Weingarten and Zehna "Estimating Reliability after Corrective Action." Management Science, Vol. 10, No. 4, July 1964.

Unclassified
Security Classification

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)* U. S. Naval Postgraduate School Monterey, California	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE

Conditional Distribution of True Reliability After Corrective Action

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

Task Progress, June 1965 - December 1965

5. AUTHOR(S) *(Last name, first name, initial)*

Larson, Harold J.

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
15 January 1966	14	1

8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S)
b. PROJECT NO. 46058	TR-61
c. Task Assignment 88423	9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*
d. Sub-Project 2	

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11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	Special Projects Office SP-114

13. ABSTRACT

N items have been tested. Attempts are made to correct those failure modes that occur; the probabilities that these attempts are successful are assumed known. The conditional distribution of the resulting true reliability is derived and used to construct observable limits which will include the true reliability with a known probability.

DD FORM 1 JAN 64 1473

Unclassified
Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Reliability						
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